Option strategies with Linear Programming

Abstract

In practice, all option strategies are decided in advance, given the investor's belief of the stock price. In this paper, instead of deciding in advance the most appropriate hedging option strategy, an LP problem is formulated, by considering all significant Greek parameters of the Black-Scholes formula, such as *delta*, *gamma*, *theta*, *rho* and *kappa*. The optimal strategy to select will be simply decided by the solution of that model. The LP model is applied to Ericsson's call and puts options.

Keywords: Finance; Option portfolios; Linear Programming

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1. Introduction

Very often, option strategies, such as *bull spread, straddle, butterfly,* b*uy - and sell ratio spread* e t c, are decided in advance, given one's own beliefs about the future price of the underlying asset, (see for instance Levy [4]). Other traders base their strategy on one- or two particular parameters, and create "delta-neutral" and sometimes even "gamma-neutral" portfolios to obtain an arbitrage free profit, (Hull [2], Jarrow and Turnbull [3]). Obviously, if the number of parameters or constraints increases, so does the complexity of the model. As a consequence, the optimal strategy is rather knotty.

A Linear Programming (LP) formulation, as is known, can treat many constraints and still provide a quick and relatively simple solution. Thus, instead of deciding in advance, the optimal strategy, we can rely on the solution of the LP model. Another advantage with the LP solution is that we can estimate the marginal costs (shadow prices) of every constraint and examine how the arbitrage free profit changes if a given constraint is included or excluded. The investor will make the final decision. If he is rather risky, he will neglect many parameters or constraints, in order to maximize his risky profit. If he is risk averter, he will attempt to secure his smaller profit, by simply including some additional constraints.

Rendleman [6] developed a simple LP model to determine the optimal mix of securities to hold long and short, in a portfolio consisting of options and stocks as well. The LP model presented in this paper is very similar to that and is applied to Ericsson's call and put options real data, as traded on February 13th 2001, at the Stockholm Stock Exchange.

The article is organized as follows. In section two we discuss shortly hedging strategies using the Greek parameters of the Black-Scholes (BS) options pricing model. In section three we present all these parameters for Ericsson's options; in section four, we will formulate an LP model using these parameters. This unusual order simplifies matters significantly, given the complex notation of options by type (call, put, buy, sell), maturities (from a day to months) and exercise prices (from very low to very high). The solution of the model is presented in section 5. Finally, in section 6, we present some concluding remarks.

2. Hedging using the Greek parameters of Black-Scholes

As is well known, the Black-Scholes [1] formula for call options (see Hull [2] for a thorough analysis) is: $C = SN(d_1) - Ee^{-rt}N(d_2)$

$\delta = N(d_1)$, where $d_1 = \dfrac{\ln(\frac{S}{E}) + \left[r + (\frac{\sigma^2}{2}) \right] t}{\sigma\sqrt{t}}$, and $d_2 = d_1 - \sigma\sqrt{t}$

N (d) is the probability that a normally distributed stochastic variable *R* will be lower or equal to *d*; σ is the stock's annualised standard deviation (volatility), S is the stock price, E is the exercise price, r is the risk free interest rate and t is time to maturity.

This equation is valid if no dividends are paid over this period and the option is of European type. Using the call-put parity we can price put options too.

The derivatives, called the Greek parameters, of the BS model have been also analysed extensively in Finance, (Hull [2], Jarrow and Turnbull [3]).

Very often, option traders are more interested in portfolio's delta ($\frac{\partial BS}{\partial S}$) and less in the stocks trend. Thus, if we are interested in a risk free arbitrage profit, we should set the portfolio's delta equal to zero. For instance, if delta is 0.6 and the stock price is SEK 80, delta neutrality implies that we can sell a call option and buy 0.6 of the stock, i.e. neutralize our exposition for small changes in stock prices, at a cost of SEK 48. If, one the other hand, we wish to hold a more aggressive portfolio, the portfolio's delta must differ from zero. Thus, the simplest way to start with, is to construct a "delta-neutral" portfolio.

Moreover, it is not sufficient to set the delta value equal to zero, because that is valid for small changes in stock price. When the changes are larger, a delta-neutral hedge cannot capture the non-linear relationship between changes in option and stock prices. The non-linear relationship implies changes in delta as well. In order to maintain delta-neutrality we are then forced to make continuous portfolio adjustments, which are obviously expensive. A method that avoids frequent portfolio adjustments and minimizes the risk that larger stock changes cannot be delta-neutralized is to set the portfolio's gamma ($\frac{\partial^2 BS}{\partial S^2}$), equal to zero. The following simple example illustrates how delta- and gamma-neutral portfolio minimizes losses, compared to just a delta-neutral one.

Assume S = SEK 840, t = 2 months, r = 4 % and σ = 50 %. Simple calculations yield the following BS call prices, delta and gamma, (Table 1 below). The first two rows show the given BS prices, while the third row depicts the price of the call option if the stock price jumped to SEK 880.

Table 1
Some BS parameters

E	Call price	Delta	Gamma
720	177.83	0.81	0.0016
820	80.74	0.6	0.0023
S_1 = 880	106.46		

A delta-neutral investor needs to make two simple transactions; (a) sell the call option for SEK 80.74, and (b) purchase shares for SEK 0.6*840 = 504. If the stock price increases to SEK 880, other things being equal, the investor's portfolio increases by SEK 0.6*(880-840) = 24 and decreases by SEK 106.46 – 80.74 = 25.72, leading to a loss of 1.72, which in fact, implies a non-hedged portfolio.

That portfolio can be hedged by delta- and gamma-neutrality, applying the following transactions; (a) buy 0.0023 / 0.0016 = 1.44 call options with E = 720; (b) sell a call option with E = 820, and (c) sell –0.6 + 1.44*0.81 = 0.57 shares at the current price, S = SEK 840.

As one can notice, the gamma-neutral portfolio's delta value is 0.57 and not 0.6 as in the previous case. The value of call options is: 1.44*(177.83 – 144.15) – (106.46 – 80.74) = SEK 22.76. Similarly, the value of stocks is: SEK -0.57*(880-840) = -22.80, i.e. almost a zero loss.

Gamma-neutrality protection against large changes in stock prices relies on constant volatility and does not protect against changes in that parameter. If for instance we estimate the stock's volatility in order to calculate the call - and put options, we run the risk that the market's implicit volatility might be different than ours. Even if these estimates coincide for a moment, they might be revised after a while, due to changes in the stock price and the passage of time. This risk can be therefore controlled if kappa ($\frac{\partial BS}{\partial \sigma}$) is set equal to zero. Hull [2] argues that, in general, a gamma-neutral portfolio does not imply kappa-neutrality and vice versa.

As time passes, it is difficult to keep gamma-neutrality, unless one is able to adjust his positions in line with the transactions made above. Otherwise, a theta-neutral ($\frac{\partial BS}{\partial t}$) portfolio is also required. Obviously theta-neutrality is different from the previous neutralities, because the passage of time is certain. Delta-neutral traders, who do not rely on gamma-neutrality, often consider theta-neutrality as a good approximation of gamma. Finally, the rho-neutrality ($\frac{\partial BS}{\partial r}$) neutralizes the interest rate risk.

Since the number of transactions increases and the positions must be revised when the number of parameters increases, it would be rather difficult to find out the optimal portfolio without the aid of specific Finance packages or LP models similar to that presented in section 4.

3. Ericsson's Greek parameters

On February 13th 2001, one hour before the Stockholm Stock Exchange was closed, Ericsson's various call - and put options prices were observed. Many call and put contracts were in fact traded over the last hour. Meanwhile, the stock price was trading at SEK 96.

Given these market prices, the risk free interest rate of 4 %, the number of days to expire 66 (for April options) and 122 (for June options), the Black-Scholes implicit volatility was estimated to be 57 % for April options and 55 % for June options, irrespective of the exercise prices. On the other hand, the three and six months historic volatilities were almost 68 % and 65 % respectively If we use these two historic parameters we can estimate the theoretic prices of April and June options. Table 2 depicts all interesting parameters for Ericsson, based on the values mentioned above. All calculations were run in Mathematica.[1]

P_{market} are final prices paid for these options, while P_{theor} are the theoretical prices one should pay, given all relevant parameters and of course the historic volatilities. All theoretic prices are rounded to the nearest SEK 0.05. Gamma and kappa for both call - and put options are identical, because the symmetry in normal distribution implies that $N(-d) = 1 - N(d)$.

Moreover, these estimates are not in accordance with the volatility smile argument, Hull [2]. According to some researchers, the implied volatility is not constant, but it decreases as the exercise price increases and kappa should be calculated from a model in which volatility is stochastic. This pattern, called "crashophobia", was observed after the stock market crash of

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[1] Observe that time is measured backwards. High t implies long time left to expire, and t = 0 implies expire date. Very often, theta is defined as $\frac{\partial(-BS)}{\partial t}$, which is obviously negative. In our estimates, we defined theta as negative.

Table 2
The Greeks and the theoretical prices of Ericsson's options

E	P_{market}	P_{theor}	delta	gamma	theta	rho	kappa
Call options (April, σ = 0.68)							
95	10.25	11.80	0.5815	0.01407	-31.741	7.9607	15.944
100	7.75	9.65	0.5118	0.01436	-32.187	7.1295	16.279
105	6	7.85	0.4442	0.01423	-31.714	6.2918	16.126
110	4.50	6.35	0.3816	0.01373	-30.472	5.4794	15.563
115	3.10	5.10	0.3246	0.01295	-28.655	4.7155	14.685
120	2.50	4.05	0.2736	0.01199	-26.433	4.0153	13.586
Call options (June, σ = 0.65)							
110	7.5	9.75	0.4448	0.01095	-22.640	11.008	21.929
115	6.5	8.35	0.3986	0.01070	-22.025	9.9906	21.422
120	4.75	7.15	0.3556	0.01032	-21.181	9.0156	20.674
Put options (April, σ = 0.68)							
95	8	10.10	-0.4185	0.01407	-27.969	-9.0936	15.944
100	10.75	12.90	-0.4887	0.01436	-28.216	-10.822	16.279
105	14.25	16.10	-0.5558	0.01423	-27.544	-12.558	16.126
110	17	19.50	-0.6184	0.01373	-26.108	-14.268	15.563
115	21	23.25	-0.6754	0.01295	-24.088	-15.929	14.685
120	25	27.20	-0.7264	0.01199	-21.668	-17.527	13.586
Put options (June, σ = 0.65)							
110	20	22.30	-0.5552	0.01095	-18.299	-25.271	21.929
115	22.25	25.85	-0.6014	0.01070	-17.486	-27.937	21.422
120	26.75	29.55	-0.6444	0.01032	-16.445	-30.561	20.674

October 1987 and perhaps was still valid thirteen years later, though not likely. Despite the fact that some studies, Hull [2], show that stochastic volatilities seem to provide similar results to those obtained from constant volatilities, new volatility parameters with a small decline were used. We started with .68 for April options with exercise price 95 and ended with .63 for April with exercise price 120. Similarly, we started with .65 for June 110 and ended with .63 for June 120. Obviously, when these kappa values were used, the theoretic option prices, as well as all Greek parameters, were affected. In Appendix A, (Table A1) we present all new parameters.

4. An LP formulation

The following notations apply: ***K*** denote buy, ***S*** denote sell, ***C*** is the call option, ***P*** is the put option, ***E*** is the exercise price, ***A*** is April and ***J*** is June. For instance, ***KC95A*** means buy a call option with exercise price SEK 95 and expiration date April, while ***SP115J*** means sell a put option with exercise price 115 and expiration date June. There are 36 variables, i.e. 18 for call options (9 for ***K*** and 9 for ***S*** for all these ***E***) and equally as many for put options. Simple capital letters ***K*** and ***S*** are used for buying and selling the stock respectively.

From the discussion in section two it is clear that our purpose is to construct a hedged option portfolio, by taking into account all other Greek parameters and the size (scale) of portfolio.

First of all, as was mentioned earlier, no a priori strategies will be considered, or all existing strategies and combinations of them are possible. Second, to simplify the formulation, we disregard all transaction costs that are associated with options trade. Obviously, if these costs were included, the value of the optimal portfolio found, would be reduced substantially. Hull [2] argues though that there are big economies of scale for a large portfolio of options, because the cost of daily adjustments is covered by the profit on many different trades. Thus, this very simple model that is used for a single stock, could be applied to larger option portfolios, consisting of many stocks and other securities.

It is clear from Table 2, that all theoretical prices are higher than the market prices for all options. This difference indicates that (theoretical) profits are possible if the market prices are adjusted to their theoretical values[2]. How quick that adjustment would take place, depends on the model used. In BS model for instance (of European type), it will occur at expire. During that period, one can make an arbitrage free profit, if he picks up a position now and an opposite position at expire, no matter where the stock price would end that day. Delta neutrality ensures such a position.

When one buys an option (in our example no matter if it is call or put) to the market price, he will earn the difference between the theoretical and market price, when this gap disappears. Similarly, to issue call or put options would lead to lower premia received than the theoretical ones. As a consequence, the objective function is the sum of differences between the theoretical and market prices for all options. This is easily formulated as:

Max $(11.8 - 10.25)KC95A + ... + (7.15 - 4.75)KC120J - (11.8 - 10.25)SC95A - ... - (7.15 - 4.75)SC120J + (10.1 - 8)KP95A + ... + (29.55 - 26.75)KP120J - (10.1 - 8)SP95A - ... - (29.55 - 26.75)SP120J$

Notice that the signs of identical parameters alter from plus, when the specific option is bought, to minus, when the same option is issued.

Let us now formulate the neutrality constraints we discussed in section two.

Delta-neutrality is achieved when the sum over all deltas plus the number of shares purchased minus the number of shares sold is equal to zero. It is easily formulated as:

$$0.5815KC95A + ...+ 0.3556KC120J - 0.5815SC95A - ... - 0.3556SC120J - 0.4185KP95A - ... - 0.6444KP120J + 0.4185SP95A + ...+ 0.6444SP120J + K - S = 0 \quad (1)$$

Similarly, gamma-neutrality is achieved if all gammas are equal to zero. Since gamma neutrality constraint appears together with delta-neutrality, we do not include the share's position in gamma-neutrality. This constraint is formulated simply as:

$$0.01407KC95A + ...+ 0.01032KC120J - 0.01407SC95A - ... - 0.01032SC120J + 0.01407KP95A + ...+ 0.01032KP120J - 0.01407SP95A - ... - 0.01032SP120J = 0 \quad (2)$$

Notice that buying call - or put options has positive gammas, while selling these options has negative gammas. It is clear that, the constraint remains unchanged if these signs are exchanged. Only if all gammas had the same sign would lead to infeasible solution.

We continue with theta-, rho- and kappa-neutrality, i.e.:

[2] A simple strategy in this case was to buy straddle in large volumes. That would cost significantly though, because one had to pay for both calls and puts. In addition, the risk with that strategy would be very high if the stock price remained unchanged.

$$-31.714KC95A - ... - 21.181KC120J + 31.714SC95A + ...+ 21.181SC120J - 27.969KP95A - ... - 16.445KP120J + 27.969SP95A + ...+ 16.445SP120J = 0 \quad (3)$$

$$7.9607KC95A + ...+ 9.0156KC120J - 7.9607SC95A - ... - 9.0156SC120J - 10.822KP95A - ... - 30.561KP120J + 10.822SP95A + ...+ 30.561SP120J = 0 \quad (4)$$

$$15.944KC95A + ...+ 20.674KC120J - 15.944SC95A - ... - 20.674SC120J + 15.944KP95A + ...+ 20.674KP120J - 15.944SP95A - ... - 20.674SP120J = 0 \quad (5)$$

Observe that the structure of signs in constraints (2) and (5) are similar. Both constraints are needed though, because the parameters differ. If we eliminate one of them, the optimal portfolio will consist of similar type of options, but with different numerical values.

It is easy to see that these constraints do not ensure a unique solution. One can repeat the same portfolio 100 times to achieve 100 times higher profit. Therefore additional constraints are required to avoid the unbounded solution.

A classic constraint to include is the budget constraint. In principle, that constraint would be formulated as the sum of all premia one receives, minus the premia he pays, minus the margins, or security it is required when the options are issued, to be equal to zero. Because the margins for different option positions vary, it would be extremely complicated to formulate an explicit budget constraint.

Another alternative is to set upper limits with all options we trade with. Of course, the lower the bounds, the lower the profit. On the other hand, in order to achieve very high profits one would need to buy many options (very high limits) and that would influence their prices.

A third alternative is to include a scale constraint to decide the relative size in the optimal solution. For instance, one can think of as the sum of buying call options delta, of selling put options delta, and of buying the share (K) should not exceed 1,000 shares. Because the portfolio's total delta is zero, the selling of shares (S) will be limited to 1,000 too. Such a scale constraint can be formulated as:

$$0.5815KC95A+ ...+ 0.3556KC120J + 0.4185SP95A + ...+ 0.6444SP120J + K \leq 1{,}000 \quad (6)$$

The problem is now formulated and a solution exists (given of course that all variables are non-negative).

Moreover, let us think of some other possible constraints that can be included, to consider three problems that might appear.

(a) Given the fact that options and shares are often traded in discrete numbers (often in 10 option contracts) all variables should be formulated as integer.
(b) Since the formulation above does not ensure that one can buy and sell simultaneously the same call or put option, we must eliminate such irrelevant transactions. We need therefore new binary [0,1] variables for buying call options (YKC), for selling call options (YSC), for buying put options (YKP) and for selling put options (YSP), for all exercise prices. For instance, in order to eliminate buying and selling April 95 call options, these constraints can be formulated as:

$YKC95A + YSC95A \leq 1$ (i)
$KC95A \leq M{*}YKC95A$ (ii)
$SC95A \leq M{*}YSC95A$ (iii)

Where, M stands for a very large number.

Constraint (i) allows both binaries to be zero, or at most one of them to be equal to one. Which of them will be equal to one, is decided by constraints (ii) and (iii).

(c) The fact that all Greek derivatives are equal to zero, does not hedge perfectly, if for instance a dramatic stock exchange collapse[3] takes place. As was mentioned in section two, delta and gamma derivatives catch only small changes of the stock price. One can therefore think of setting explicit bounds on the share and on the ratio of puts to calls. Such a constraint between selling of call options, buying of call options and buying the share can be formulated as:

$$\frac{SC95A+SC100A+...+SC120J}{KC95A+KC100A+...+KC120J+K} \leq N \qquad \text{(iv-a)}$$

Where, N stands for an arbitrary ratio, such as 2.

A similar constraint (iv-b) can be formulated regarding selling of put options, buying of put options and selling the share. Notice that when these two constraints are included in the formulation above, we increase the probability that the problem (b) appears, which forces us to include constraints (i)-(iii) as well.

5. Results

The problem was solved twice; (i) using the parameters on Table 2; (ii) using the parameters on Table A1 (see Appendix). Below we present the estimates based on (i). The estimates based on (ii) are presented in Appendix (Table A2).

We solved first the problem in its simple version, i.e. including the first six constraints, (model 1), without taking into account (a), (b) and (c) modifications. Thereafter we dropped one of the Greek constraints at a time, but always kept the delta and scale constraints, to see how the profit varies. Because in the optimal solution we bought and sold various call and put options, but with different exercise price, we did not have to consider the (b) formulation. The solution remained unchanged even when (c) was included. On the other hand, when in the initial formulation all variables were treated as integer, the solution was changed and the profit, as expected, decreased significantly.

Table 3 summarizes the optimal solution for all models. The constraints for every model are depicted in the first column. The second column shows the respective shadow price, the third column shows the optimal position and the last column the respective profit. Observe that the formulation assumes one option is equivalent to one share. To obtain the equivalent amount of shares, these values must therefore be divided by 100. The integer solution is also shown at the bottom of the table.

[3] By the middle of March, Ericsson (as all other shares and especially the IT shares) plumbed to SEK 60, and on October 2002 to less than SEK 6!

Table 3
Optimal portfolio with Ericsson's options

model	Shadow price	Transaction and exercise price			max profit
delta	0	buy	6.87 call	April 115	2,181.33
gamma	999.76	buy	153.545 call	June 120	
theta	0.4030	buy	1,584.207 put	June 115	(1)
rho	-0.00301	sell	6.976 put	April 105	
kappa	-0.00626	sell	1,691.809 put	June 110	
scale	2.1813	sell	47.25 shares		
delta	0	buy	157.813 call	June 120	2,186.24
gamma	57.357	sell	0.737 call	April 95	
rho	0.07229	buy	1,588.556 put	June 115	(2)
kappa	-0.05145	sell	1,700.075 put	June 110	
scale	2.1862	sell	44.2 shares		
delta	-1.907	buy	118.043 call	June 120	2,287.154
gamma	34.752	buy	1,662.787 put	June 115	
kappa	0.09715	sell	108.186 put	April 105	(3)
scale	2.2871	sell	1,623.835 put	June 110	
delta	-2.7036	buy	402.12 call	June 120	4,098.545
gamma	184.491	buy	1,662.787 put	June 115	
scale	4.0985	sell	1,541.932 put	April 105	(4)
delta	-5.9860	buy	2,812.148 call	June 120	12,735.2
scale	12.7351	buy	1,662.787 put	June 115	(5)
		Integer solution			
delta	-	buy	21 call	April 115	661.5
gamma		buy	1 call	April 120	
theta		buy	187 call	June 120	
rho		sell	10 call	April 95	
kappa		sell	14 call	April 120	(6)
scale		sell	46 call	June 115	
		buy	8 put	April 95	
		buy	819 put	June 110	
		buy	160 put	June 115	
		buy	591 put	June 120	
		sell	6 put	April 95	
		sell	1,664 put	June 110	
		sell	37 shares		

As is seen, the integer solution is different and was obtained after 15,087 iterations. In addition it suffers from problem (b) since we should buy and sell similar calls or puts. When the integer problem was reformulated by taking into account the constraints (i)-(iii), no integer solution was found.

When the variables were treated as continuous, as expected, the more constraints we impose on the model, the lower (but more certain) the profit. To exclude theta and even rho, makes no big difference, because the profit increases marginally. On the other hand, if we exclude kappa and gamma, we increase the profit (and the risk as well) significantly.

When all Greek derivatives are included, (model 1), the optimal solution reveals the following strategy (excluding the two small April options). We should buy call options June 120 equivalent to 154 shares (i.e. 1.54 call options), buy put options June 115 equivalent to 1,584 shares (i.e. 15.84 put options), sell put options June 110 equivalent to 1,692 shares (i.e. 16.92 put options), and sell almost 47 shares.

The shadow price of the delta constraint is zero, meaning that the profit does not increase if we increase the delta risk by one unit, such as by selling one more share[4]. If the shadow price is negative, as in models 3, 4 and 5, the profit will increase if we reduce the right hand side by one unit. That is consistent with models 1 and 2, where we sell a specific amount of shares, but not with models 3, 4 and 5. In those models we have already sold out all shares and increased the profit already.

The other Greek shadow prices (but not the gamma) are very low and influence the profit marginally. Gamma, on the other hand, has a very large impact on profit. That is seen clearly in models 4 and 5. To include gamma-neutrality we decrease the profit by 67%. Kappa has also a large impact on profit. The scale constraint's shadow price is rather constant, when at least two other Greek derivatives are included. The profit would increase by almost SEK 2, for every new share in the portfolio.

If we now return to the general model 1, and exclude the two small April options, we would receive (with no transaction costs) SEK 2,376 on the 13th of February. [5]

Let us now check the value of this portfolio, at expire in June, for various stock prices. Table 4 summarizes the payoff of this portfolio (excluding the two small April options). The fifth raw shows the portfolio value, based on the respective number of shares from model 1, i.e. 154, 1,584 and 1,692. For instance, if the stock price ended at SEK 80, the payoff would be equal to the following:

The K 120 calls would be out of money. The K 115 puts would be equal to 115 – 80 = 35* 1,584 = 55,440. The S 110 puts would be equal to 80 – 110 = -30*1,692 = -50,760. The 47 shares that were sold in February at SEK 96 should be bought back and a cost of 3,760. Thus, the value would be 55,440 – 50,760 – 3,760 = SEK 920.

Table 4
Value of portfolio based on June options for various stock prices

	54[6]	70	80	90	100	110	120	130
K 120 call	0	0	0	0	0	0	0	10
K 115 put	61	45	35	25	15	5	0	0
S 110 put	-56	-40	-30	-20	-10	0	0	0
47 Shares	-2,538	-3,290	-3,760	-4,230	-4,700	-5,170	-5,640	-6,110
Sum (June)	-666	310	920	1,530	2,140	2,750	-5,640	-4,570
Net	1,710	2,686	3,296	3,906	4,516	5,126	-3,264	-2,194

[4] The right hand side of the first constraint is equal to zero. If we move -S to the right, it gets +S. If we increase S by one unit, the profit remains unchanged.

[5] Paid premium: K 120 call = 4.75*154 = 731.5 and K 115 put = 22.25*1,584 = 35,244
Received premium: S 110 put = 20*1,692 = 33,840 and 47 shares*96 = 4,512

[6] The stock price at expire in June was SEK 54.

Finally, the last raw shows the net value of the portfolio, by including what we received in February. For the same column its value is: 920 + 2,376 = SEK 3,296

It is clear that positive profits are more frequent. Negative profits would have brought about if the stock price increased from SEK 96 to almost 120 and up to approximately 150. Thereafter, mainly due to K 120, profits would appear again. The highest profits would occur if the stock price increased at a moderate path, up to SEK 110. Profits would have been made even if the stock price declined dramatically (as it did). These profits are very small indeed, or non-existent if the transactions costs are taken into account.

Lastly, a glance at Appendix (Table A2) shows that, as expected, the profits are lower, while the optimal option portfolios do not differ significantly from those on Table 3. Indeed, in some cases, there are almost identical.

6. Concluding remarks

LP formulations simplify significantly complicated problems like the optimal strategy for portfolio of options. The optimal strategy is simply the solution of the LP model, irrespective of one's subjective beliefs. But even if one was able to decide in advance the correct transactions, it would be almost impossible to find out the precise amount of calls, puts and shares that should be purchased and/or sold, without the aid of computers or models like the above. In addition, the shadow prices from such an LP problem will help the trader/investor to find out if, for instance, the kappa-neutrality is worthwhile to consider and how it influences the profit if we increase the kappa risk by one unit.

On the other hand, LP formulations have their limitations. For instance, by applying linear constraints to such financial derivatives we might obtain a false solution, if the true formulation is non-linear and some of these derivatives, such as kappa, are neither constant nor deterministic. Our estimates show that the optimal solutions differ, not significantly though, if the volatility is characterised by a small volatility smile. A second limitation was the absence of all transaction costs and margins required with options trade, which will force the trader/investor to minimize his transaction, by focusing on rather risky delta- and gamma-neutral portfolios. Thus, further research is needed in order to evaluate the correct impact of all Greek parameters.

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Appendix

Table A1
The Greeks and the theoretical prices of Ericsson's options

E	P_{market}	P_{theor}	delta	gamma	theta	rho	kappa
Call options (April, σ = 0.68, 0.67, 0.66, 0.65, 0.64, 0.63)							
95	10.25	11.80	.5815	.01407	-31.741	7.9607	15.944
100	7.75	9.50	.5098	.01458	-31.742	7.1314	16.280
105	6	7.50	.4391	.01463	-30.761	6.2609	16.095
110	4.50	5.85	.3714	.01424	-28.929	5.3850	15.431
115	3.10	4.50	.3082	.01347	-26.427	4.5359	14.364
120	2.50	3.40	.2508	.01237	-23.461	3.7398	12.993
Call options (June, σ = 0.65, 0.64, 0.63)							
110	7.5	9.75	.4448	.01095	-22.640	11.008	21.929
115	6.5	8.15	.3947	.01083	-21.646	9.9402	21.366
120	4.75	6.75	.3468	.01056	-20.372	8.8733	20.489
Put options (April, σ = 0.68, 0.67, 0.66, 0.65, 0.64, 0.63)							
95	8	10.10	-.4185	.01407	-27.969	-9.0936	15.944
100	10.75	12.80	-.4902	.01458	-27.771	-10.818	16.280
105	14.25	15.75	-.5609	.01463	-26.592	-12.586	16.095
110	17	19.10	-.6286	.01425	-24.561	-14.359	16.431
115	21	22.65	-.6918	.01347	-21.860	-16.103	14.364
120	25	26.50	-.7492	.01238	-18.698	-17.800	12.993
Put options (June, σ = 0.65, 0.64, 0.63)							
110	20	22.30	-.5552	.01095	-18.299	-25.271	21.929
115	22.25	25.60	-.6053	.01084	-17.107	-27.987	21.366
120	26.75	29.15	-.6532	.01056	-15.636	-30.703	20.489

Table A2
Optimal portfolio with Ericsson's options

Model	Transaction and exercise price			Profit
Delta	buy	120.95 call	April 115	1779.6
gamma	buy	174.73 call	June 120	
theta	buy	1,415.25 put	June 115	(1)
rho	buy	147.80 put	April 95	
kappa	sell	1,558.38 put	June 110	
scale	sell	324.92 call	April 120	
Delta	buy	191.52 call	June 110	1787.6
gamma	sell	150.58 call	April 120	
rho	buy	1,517.10 put	June 115	(2)
kappa	sell	1,656.79 put	June 110	
scale	buy	104.98 put	April 95	
Delta	buy	944.93 call	April 100	1912.15
gamma	buy	990.57 put	June 115	
kappa	sell	924.00 put	April 105	(3)
scale	sell	1,014.46 call	June 115	
Delta	buy	2,389.49 call	June 110	4,930.96
gamma	buy	569.19 put	June 115	
scale	sell	2,613.52 call	April 120	(4)
Delta	buy	2,883.51 call	June 120	11, 301.5
scale	buy	1,652,07 put	June 115	(5)

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